

# Geometric Transition, Large N Dualities and MQCD Dynamics

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## ABSTRACT

We study Vafa's geometric transition from a brane setup in M-theory. In this transition D5 branes wrapped on  $\mathbf{P}^1$  cycles of a resolved conifold disappear and are replaced by fluxes on a deformed conifold. In the limit of small sized  $\mathbf{P}^1$ , we describe this mechanism as a transition from curved M5 branes to plane M5 branes which replaces  $SU(N)$  MQCD by  $U(1)$  theories on the bulk. This agrees with the results expected from the geometric transition. We also discuss the reduction to ten dimensions and a brane creation mechanism in the presence of fluxes.

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# 1 Introduction

Many interesting results have been obtained by realizing gauge field theories on the world-volume of branes in string theory (for a review see [1]). One of the most interesting direction was to study aspects of confinement in the M theory fivebrane version of QCD (MQCD) [2, 3]. The brane theory (MQCD) is not identical to QCD as it contains the Kaluza-Klein modes from the compactified direction  $x^{10}$  and the radius of the direction  $x^{10}$  becomes an extra parameter. In [3], different aspects of  $\mathcal{N} = 1$  MQCD were studied, it was shown to have flux tubes and the tensions of MQCD strings and domain walls were derived.

One question which arises is the following: MQCD results were based on studying the M5 brane obtained by lifting a brane configuration with D4 branes between two orthogonal NS5 branes. The gauge theory of the latter is  $U(N)$  if we have N D4 branes whereas the gauge theory on the M5 brane is  $SU(N)$  and the two coincide in the extreme IR because the  $U(1)$  is asymptotically free and decouples, but away from IR the field theory on the M5 brane is  $SU(N)$ . We can then ask what happens to the  $U(1)$  group – do we lose all the information about it or can we recover the information in some limits? To answer that, we will inspire ourselves from the recent results about large N dualities of [5].

Recently, important results have been obtained concerning field theories on partially wrapped D-branes over non-trivial cycles of non-compact geometries. In particular, based on the Chern-Simons/topological strings duality [4], Vafa suggested that D-branes wrapped over cycles have a dual description where the D-branes have disappeared and have been replaced by fluxes after transition in geometry [5]. This duality can be explained as a geometric flop in M theory on a  $\mathbf{G}_2$  holonomy manifold [8, 7]. Other developments related to this duality in 10 and 11 dimensions were discussed in [10, 11, 6, 12, 13, 15, 16, 17, 18] and in the presence of orientifold planes (besides the D-branes) in [9, 14]. Vafa's duality considers strings and branes at conifold singularities which have been studied extensively in the recent years, starting with the work of Klebanov-Witten [19] and generalized to include quotient conifold singularities [20, 21, 22], nonconformal field theories [23, 25] and theories with non-commutative moduli spaces [26].

We will be able to motivate, from the discussions related to the brane configurations for conifolds, the important result that for N D5 branes on  $\mathbf{P}^1$  cycle of a resolved conifold, in the limit when the cycle is of very small size there is a transition from the  $U(N)$  theories on the branes to  $U(1)$  theory. This is because there is a T-dual picture for the resolved conifold which implies a configuration with N D4 branes on an interval between two orthogonal NS5 branes which can be lifted to a *single* M5 brane and is the starting

point of the MQCD discussion. What happens in M theory when the size of the  $\mathbf{P}^1$  becomes small? The result, as we shall show, is that in this case the curve M5 brane splits into a set of  $N$  “plane” M5 branes located at a specific point in the  $x^{10}$  direction. Also now on each planar M5 branes we have a  $\mathcal{N} = 1, U(1)$  field theory. From where are the  $U(1)$  groups coming from? They come exactly from the  $U(1)$  which was decoupled in the initial step of going from  $U(N)$  to  $SU(N)$ . As explained in [3], the  $U(1)$  were decoupled because it had an infinite kinetic energy. What we show now is that in the limit of small size  $\mathbf{P}^1$  cycle, the energy becomes finite so it makes sense to reconsider the  $U(1)$  groups.

This also tells us that in the limit of small  $\mathbf{P}^1$  cycle, one has to reconsider the issue of confinement and chiral symmetry breaking. In [3], the issue was treated for pure QCD and the result was that different vacua were obtained for different curved M5 branes, the theory had a mass gap and the  $N$  vacua rotated by  $Z_{2N}$ . In our case, we have condensation but still have massless particle after condensation because we have the  $U(1)$  groups on the plane M5 branes and the group  $Z_{2N}$  rotates different  $U(1)$  theories located on the different plane M5 branes. We also obtain an interpretation of the domain walls and QCD strings in the small  $\mathbf{P}^1$  limit.

We can now return and ask if our transition can be seen as an M theory lift of the large  $N$  geometrical transition. This could be an alternative to the one involving  $G_2$  manifolds [8, 7]. We see that the plane M5 branes reduce to a type IIA configuration which is T-dual to the deformed conifold with flux through the  $\mathbf{S}^3$  cycle. The fluxes are due to bending of the planar M5 branes along the  $x^{10}$  direction implying a change in metric. In 10 dimensions this reduces to a configuration of two intersecting NS5 branes with a two-form flux whose T-dual is our required configuration.

## 2 T-dual description of Vafa’s $\mathcal{N} = 1$ Duality

In type IIA string theory, a four dimensional  $\mathcal{N} = 1, U(N)$  supersymmetric gauge theory is obtained by wrapping  $N$  D6 branes on the  $\mathbf{S}^3$  in a complex deformed conifold (we will simply call it a deformed conifold):

$$xy - uv - \mu = 0 \tag{1}$$

which is isomorphic to  $T^*\mathbf{S}^3$  as a symplectic manifold after the rotation of symplectic structure by the phase of  $\mu$ . In [5], Vafa has proposed a duality, in the large  $N$  limit, between this theory and type IIA superstrings without  $D$ -branes propagating on a Kähler deformed conifold which is a small resolution of the conifold (we will call it a resolved conifold). Recall that the resolved conifold is obtained by replacing the singular point

of the conifold by a rigid  $\mathbf{P}^1$  with normal bundle  $\mathcal{O}(-1) + \mathcal{O}(-1)$ . By the definition of  $\mathbf{P}^1$ , each point  $p$  of  $\mathbf{P}^1$  represents a complex line  $L_p$  and  $\mathcal{O}(-1)$  (a.k.a. the tautological line bundle) over  $\mathbf{P}^1$  is the complex line bundle on  $\mathbf{P}^1$  with fiber  $L_p$  over  $p \in \mathbf{P}^1$ , and  $\mathcal{O}(-1) + \mathcal{O}(-1)$  is the direct sum of two copies of the  $\mathcal{O}(-1)$ . Moreover, it can be seen from the toric description of the resolved conifold that the resolved conifold is the same as the total space of the bundle  $\mathcal{O}(-1) + \mathcal{O}(-1)$  over  $\mathbf{P}^1$ .

The large  $N$  duality, which emerges from the embedding of the large  $N$  Chern–Simons/ topological string duality of Gopakumar and Vafa [4] in ordinary superstrings, states that  $\mathcal{N} = 1$   $U(N)$  theory on the deformed conifold is dual to type IIA theory on the resolved conifold. In this duality, the branes disappear and are replaced by  $N$  units of RR flux through  $\mathbf{P}^1$ , and NS flux through the fibers of the normal bundle  $\mathcal{O}(-1) + \mathcal{O}(-1)$ . The complexified Kähler parameter  $t$  of  $\mathbf{P}^1$  is related to the volume  $V$  of the  $\mathbf{S}^3$  and the string coupling constant by:

$$(e^t - 1)^N = \exp(-V/g_s) \quad (2)$$

We note that when  $\mathbf{S}^3$  has a large volume  $V$  and when the 't Hooft coupling  $Ng_{YM}^2$  is small, the Kähler parameter  $t \rightarrow 0$  and the good description is the one with wrapped D-branes, whose worldvolume decouples from the bulk and we use the open strings ending on the D-branes. When the Kähler parameter  $t \gg 0$ , the blown-up description is good and the wrapped D-brane picture is bad because then the volume of  $\mathbf{S}^3$  should be negative. In this case we use the close strings whose low-energy sector is given by supergravity. The  $SU(N)$  gauge theory decouples from the bulk when the size  $t$  of the blown-up  $\mathbf{P}^1$  is small and  $t$  is identified with the glueball superfield  $S = -\frac{1}{32\pi^2} Tr W_\alpha W^\alpha$  of the  $SU(N)$  theory, its expectation value corresponding to gaugino condensation in the  $SU(N)$  theory.

In the mirror description, the  $U(N)$  theory is obtained from type IIB D5 wrapped on the rigid  $\mathbf{P}^1$  in the resolved conifold and, in the large  $N$  limit, this is equivalent to type IIB on the deformed conifold (1) with RR flux on the  $S^3$ . The deformation parameter  $\mu$  will be identified with the  $SU(N)$  glueball superfield  $S$ . Rather than the  $N$  original D5 branes, there are now  $N$  units of RR flux through  $S^3$ , and also some NS flux through the fiber of the cotangent bundle  $T^*\mathbf{S}^3$ . Moreover, by integrating the holomorphic three form over the three cycle  $\mathbf{S}^3$  and its dual cycle in  $T^*\mathbf{S}^3$ , one obtains the superpotential

$$W_{\text{eff}} = S \log (\Lambda^{3N}/S^N) + NS, \quad (3)$$

and the  $N$  vacua of  $SU(N)$   $\mathcal{N} = 1$  supersymmetric Yang-Mills:

$$\langle S \rangle = \exp(2\pi i k/N) \Lambda^3. \quad (4)$$

Hence the large  $N$  duality arises from a geometric transition from the resolved conifold to the deformed conifold. We now take T dual of this geometric transition and later we

will consider the lifting to the M-theory. We begin by introducing a circle action on the conifold and extend it to the resolved conifold and the deformed conifold in a compatible manner. Consider an action  $S_c$  on the conifold  $xy - uv = 0$ :

$$S_c : (e^{i\theta}, x) \rightarrow x, \quad (e^{i\theta}, y) \rightarrow y, \quad (e^{i\theta}, u) \rightarrow e^{i\theta}u, \quad (e^{i\theta}, v) \rightarrow e^{-i\theta}v, \quad (5)$$

The orbits of the action  $S_c$  degenerates along the union of two intersecting complex lines  $y = u = v = 0$  and  $x = u = v = 0$  on the conifold. Now, if we take a T-dual along the direction of the orbits of the action, there will be NS branes along these degeneracy loci as argued in [24]. So we have two NS branes which are spaced along  $x$  (i.e.  $y = u = v = 0$ ) and  $y$  directions (i.e.  $x = u = v = 0$ ) together with non-compact direction along the Minkowski space which will be denoted by  $NS_x$  and  $NS_y$ .

This action can be lifted to the resolved conifold. To do that, we consider two copies of  $\mathbf{C}^3$  with coordinates  $Z, X, Y$  (resp.  $Z', X', Y'$ ) for the first (resp. second)  $\mathbf{C}^3$ . Then  $\mathcal{O}(-1) + \mathcal{O}(-1)$  over  $\mathbf{P}^1$  is obtained by gluing two copies of  $\mathbf{C}^3$  with the identification:

$$Z' = \frac{1}{Z}, \quad X' = XZ, \quad Y' = YZ. \quad (6)$$

The  $Z$  (resp.  $Z'$ ) is a coordinate of  $\mathbf{P}^1$  in the first (resp. second)  $\mathbf{C}^3$  and others are the coordinates of the fiber directions. The blown-down map from the resolved conifold  $\mathbf{C}^3 \cup \mathbf{C}^3$  to the conifold  $\mathcal{C}$  is given by

$$x = X = X'Z', \quad y = ZY = Y', \quad u = ZX = X', \quad v = Y = Z'Y'. \quad (7)$$

From this map, one can see that the following action  $S_r$  on the resolved conifold is an extension of the action  $S_c$  (5):

$$S_r : (e^{i\theta}, Z) \rightarrow e^{i\theta}Z, \quad (e^{i\theta}, X) \rightarrow X, \quad (e^{i\theta}, Y) \rightarrow e^{-i\theta}Y \\ (e^{i\theta}, Z') \rightarrow e^{-i\theta}Z', \quad (e^{i\theta}, X') \rightarrow e^{i\theta}X', \quad (e^{i\theta}, Y') \rightarrow Y' \quad (8)$$

The orbits degenerates along the union of two complex lines  $Z = Y = 0$  in the first copy of  $\mathbf{C}^3$  and  $Z' = Y' = 0$  in the second copy of  $\mathbf{C}^3$ . Note that these two lines do not intersect and in fact they are separated by the size of  $\mathbf{P}^1$ . Now we take T-dual along the orbits of  $S_r$  of type IIB theory obtained by wrapping  $N$  D5 branes on the rigid  $\mathbf{P}^1$ . Again there will be two NS branes along the degeneracy loci of the action: one NS brane, denoted by  $NS_X$ , spaced along  $X$  direction (which is defined by  $Z = Y = 0$  in the first  $\mathbf{C}^3$ ) and the other NS brane, denoted by  $NS_{Y'}$ , along  $Y'$  direction (which is defined by  $Z' = X' = 0$  in the second  $\mathbf{C}^3$ ). Therefore the T-dual picture will be a brane configuration of D4 brane along the interval with two NS branes in the ‘orthogonal’ direction at the ends of the the interval. Here the length of the interval is the same as the size of the rigid  $\mathbf{P}^1$ . As the rigid  $\mathbf{P}^1$  shrinks to zero, the size of the interval goes to zero and  $NS_X$  (resp.  $NS_{Y'}$ ) approaches to  $NS_x$  (resp.  $NS_y$ ) of the conifold.

Finally we will provide a circle action of the deformed conifold and a T dual picture under this action. Consider the following circle action  $S_d$

$$(e^{i\theta}, x) \rightarrow x, \quad (e^{i\theta}, y) \rightarrow y, \quad (e^{i\theta}, u) \rightarrow e^{i\theta}u, \quad (e^{i\theta}, v) \rightarrow e^{-i\theta}v, \quad (9)$$

on the deformed conifold

$$xy - uv = \mu \quad (10)$$

Then  $S_d$  is clearly the extension of  $S_k$  (8) and the orbits of the action degenerate along a complex curve  $u = v = 0$  on the deformed conifold. If we take a T-dual of the deformed conifold along the orbits of  $S_k$ , we obtain a NS brane along the curve  $u = v = 0$  with non-compact direction in the Minkowski space which is given by

$$xy = \mu \quad (11)$$

in the x-y plane. Topologically, the curve (11) is  $\mathbf{R}^1 \times \mathbf{S}^1$ . In the T-dual picture, the large N duality is achieved via a transition from the brane configuration of N coincident D4 branes between two ‘orthogonal’ NS branes to the brane configuration of a single NS brane wrapped on  $\mathbf{R}^1 \times \mathbf{S}^1$  with gauge fields  $A_\mu$  (which will be discussed in section 6).

### 3 MQCD Transition

To investigate the large  $N$  limit for a small  $\mathbf{P}^1$ , we appeal to Witten’s MQCD M5 brane of the brane configuration of the resolved conifold constructed in the previous section.

In MQCD [3], the classical type IIA brane configuration turns into a single fivebrane whose world-volume is a product of the Minkowski space  $\mathbf{R}^{1,3}$  and a complex curve in a flat Calabi-Yau manifold

$$M = \mathbf{C}^2 \times \mathbf{C}^*. \quad (12)$$

Recall that the T-dual brane configuration of  $N$  D5 branes wrapped on the rigid  $\mathbf{P}^1$  in the resolved conifold is the N D4 branes on the interval together with two NS branes  $NS_X, NS_{Y'}$  at the ends. We denote the coordinate of the interval by  $x^7$  and the angular coordinate of the circle  $\mathbf{S}^1$  in the 11-th dimension by  $x^{10}$ . After we combine them into a complex coordinate

$$t = \exp(-R^{-1}x^7 - ix^{10}) \quad (13)$$

where  $R$  is the radius of the circle  $\mathbf{S}^1$  in the 11-th dimension, the world-volume of the M-theory fivebrane (a.k.a. M5 brane), corresponding to the brane configuration of the

resolved conifold, is given  $R^{1,3} \times \Sigma$  where  $\Sigma$  is a complex curve defined by, up to an undetermined constant  $\zeta$

$$y = \zeta x^{-1}, \quad t = x^N \quad (14)$$

in  $M$ . Here we are using  $x, y$  instead of  $X, Y'$  anticipating the identification after the transition and we will call  $\Sigma$  a M5 curve. The equations (14) should be understood as an embedding of the punctured  $x$  plane  $\mathbf{C}^*$  into the Calabi-Yau space  $M$  by the map

$$\mathbf{C}^* \longrightarrow \Sigma \subset M, \quad x \rightarrow (x, \zeta x^{-1}, x^N) \quad (15)$$

Hence  $\Sigma$  is a rational curve and wraps around the punctured  $t$  plane  $\mathbf{C}^* \subset M$   $N$  times which reflects the fact that there are  $N$  coincident D4 branes along the  $x^7$  direction in Type IIA.

Now if we consider the limit where the size of  $\mathbf{P}^1$  goes to zero, then the  $x^7$  direction in the M-theory will be very small and negligible. The modulus of  $t$  on  $\Sigma$  will be fixed i.e.  $\Sigma$  will be a curve in the cylinder  $\mathbf{S}^1 \times \mathbf{C}^2$  where  $\mathbf{S}^1$  is the circle in the 11-th dimension and  $\mathbf{C}^2$  are coordinatized by  $x, y$ . In fact, the value of  $t$  on  $\Sigma$  must be constant because  $\Sigma$  is holomorphic and there is no non-constant holomorphic map into  $\mathbf{S}^1$ . The holomorphicity is required because of supersymmetry. Therefore the M5 curve make a transition from a “space” curve into a “plane” curve. From (14), we obtain two relation on  $t$  and  $t^{-1}$

$$t = x^N, \quad t^{-1} = \zeta^{-N} y^N. \quad (16)$$

So there are  $N$  possible plane curves which the M5 space curve  $\Sigma$  can be reduced to:

$$\Sigma_k : \quad t = t_0, \quad xy = \zeta \exp 2\pi i k / N, \quad k = 0, 1, \dots, N-1. \quad (17)$$

Alternatively we may consider these as  $N$  possible relations between  $x$  and  $y$  on  $\Sigma$  after eliminating  $t$  because the dimension of  $t$  on  $\Sigma$  is virtually zero and the information along  $t$  is not reliable.

Since this is the limit where  $g_s N$  is big, this degenerate M5 brane should not be considered as a M theory lift of D branes. In this limit, the gravitationally deformed background without the D-branes is the right description and this is a closed string geometric background. If one looks at the T dual picture of the deformed conifold (11), this is exactly M theory lift of the NS brane of the deformed conifold! The size of  $\mathbf{S}^3$  on the deformed conifold depends on the expectation value for the gluino condensation and, for each value of the gluino condensate we will have a different flux through the  $\mathbf{S}^3$  cycle. We may intuitively consider the plane M5 as one obtained from two intersecting M5 branes by smearing out the intersection point due to the flux from the vanished D4 branes wrapped on  $\mathbf{P}^1$ .

The plane M5 branes (17) describe  $N$  different  $U(1)$  theories with fluxes related to each other by  $\exp(2\pi i/N)$  after the  $SU(N)$  group gets a mass gap. These  $N$  different  $U(1)$ 's are just  $N$  different vacua for the same theory, each obtained for a specific choice of  $\langle S \rangle$ . The vacuum expectation values of the glueball superfield  $S$  are then read to be:

$$\langle S \rangle = \exp(2\pi i k/N) \zeta \quad (18)$$

which agrees with the field theory result since  $\zeta \sim N\Lambda^3$  and also with [32, 34, 35]. The  $\mathcal{N} = 2$  vector multiplet consists of a neutral  $\mathcal{N} = 1$  chiral superfield and  $\mathcal{N} = 1$  photon. The  $\mathcal{N} = 1$  chiral superfield gets a mass due to the presence of fluxes and is identified with  $S$ . So in MQCD, the large  $N$  duality occurs via a geometric transition from the space M5 curve to the plane M5 curve. The 11-dimensional supergravity solutions for this plane M5 curve have been studied in [28].

Before discussing the gauge theory on the plane M5 branes, let us consider the NS 3-form flux which goes through the non-compact cycle dual to the compact  $\mathbf{S}^3$  cycle of the deformed conifold. After the T-duality, the  $\mathbf{S}^3$  cycle translates into the waist of the NS brane  $xy = \zeta$ , considered as a hyperboloid. The NS flux will be given by an integral of the holomorphic 1 form  $dz/z$  over a noncompact 1-cycle dual to the waist. As in [5], the NS flux controls the size of the cycle that the D5 branes are wrapped on, in our case the cycle being  $\mathbf{P}^1$ . The cycle  $\mathbf{P}^1$  is a rigid one, this being the necessary condition to turn a NS flux in the geometry. The size of the  $\mathbf{P}^1$  cycle is related to the size of the interval direction of the D4 branes which is related to the gauge coupling constant of the  $SU(N)$  theory. Therefore, the NS flux controls the magnitude of the coupling constant of the  $SU(N)$  theory.

## 4 Gauge theory on M5 brane

It has been shown that M5 brane (14) gives rise to  $SU(N)$  gauge theory and  $U(1)$  gauge fields are decoupled from the theory [3]. It is puzzling to see the massless  $U(1)$  photons after the transition. To understand this phenomenon better, we recall the low energy effective four-dimensional theory from M5 brane derived in [2]. Consider in general a fivebrane whose world-volume is  $\mathbf{R}^{1,3} \times \bar{\Sigma}$  where  $\bar{\Sigma}$  is a compact Riemann surface of genus  $g$ . According to [29, 30], in the effective four-dimensional description, the zero modes of the antisymmetric tensor give  $g$  Abelian gauge fields on  $\mathbf{R}^4$ . The coupling constants and theta parameters of the  $g$  Abelian gauge fields are described by a rank  $g$  Abelian variety which is the Jacobian  $J(\bar{\Sigma})$ . Let

$$T = F \wedge \Lambda + *F \wedge *\Lambda, \quad (19)$$



where  $F$  is a two-form on  $\mathbf{R}^4$ ,  $\Lambda$  is a one-form on  $\overline{\Sigma}$ , and  $*$  is the Hodge star. This  $T$  is self dual, and the equation of motion  $dT = 0$  gives Maxwell's equations  $dF = d*F = 0$  along with the equations  $d\Lambda = d*\Lambda = 0$  for  $\Lambda$ . So  $\Lambda$  is harmonic one-form and every choice of a harmonic one-form  $\Lambda$  gives a way of embedding solutions of Maxwell's equations on  $\mathbf{R}^4$  as a solution of the equations for the self-dual three-form  $T$ .

We now consider  $n + 1$  parallel Type IIA NS branes joined by D4 branes. The M5 brane world-volume will be of the form  $\mathbf{R}^{1,3} \times \Sigma$ . The  $\Sigma$  is obtained by joining  $n + 1$  copies of  $\mathbf{C}$  by  $N_k$  tubes  $\mathbf{C}^*$  between  $k$ -th and  $(k + 1)$ -th  $\mathbf{C}$ . By adding  $(n + 1)$  points, the  $\Sigma$  can be compactified into a compact Riemann surface  $\overline{\Sigma}$  of genus  $g = \sum_{k=1}^n (N_k - 1)$  as was shown in [2]. A Jacobian (or quasi-Albanese) of a non-compact Riemann surface can be defined in terms of its mixed Hodge structure [31]. For the Riemann surface  $\Sigma$ , the Jacobian  $J(\Sigma)$  fits into an exact sequence of algebraic groups:

$$1 \longrightarrow (\mathbf{C}^*)^n \longrightarrow J(\Sigma) \longrightarrow J(\overline{\Sigma}) \longrightarrow 0 \quad (20)$$

where  $J(\overline{\Sigma})$  is the usual Jacobian of the compactification  $\overline{\Sigma}$  of  $\Sigma$ . This exact sequence does not split in general as the Jacobian  $J(\Sigma)$  represents a non-trivial elements of  $\text{Ext}(J(\overline{\Sigma}), (\mathbf{C}^*)^n)$ . In the effective action for the four dimensional gauge theory, the harmonic forms from the non-compact part  $(\mathbf{C}^*)^n$  decouples from the theory because the corresponding M5 brane kinetic energy  $\int_{R^{1,3} \times \Sigma} |T|^2$  becomes infinity as the non-trivial harmonic one form  $d \log z$  on  $\mathbf{C}^*$  is not square-integrable [2]. Thus the low energy effective action is determined by  $J(\overline{\Sigma})$  and the gauge group is  $\prod_{k=1}^n SU(N_k)$ . On the other hand, according to the large N duality proposal of [6], the gauge theory becomes  $U(1)^n$  after the transition and it agrees with  $U(1)^n$  theory from  $U(N)$  theory after the  $SU(N_i)$  gets a mass gap and confine in the breaking

$$U(N) \rightarrow \prod_{k=1}^n U(N_k). \quad (21)$$

Hence after the transition, the coupling from the non-compact part  $(\mathbf{C}^*)^n$  of the Jacobian  $J(\Sigma)$  has been restored. From the M-theory point of view, the large N duality is a transition from the compact part  $J(\overline{\Sigma})$  to the non-compact part  $(\mathbf{C}^*)^n$  of the Jacobian  $J(\Sigma)$ . So there will be a duality between  $\prod_{k=1}^n SU(N_k)$  and  $U(1)^n$  and the former will be described by the space M5 curve and the latter will be described by the plane M5 curve. In this paper we will deal with the simplest case of  $n = 1$  and more complicated case ( $n > 1$ ) will be dealt in [42]. As the  $\mathbf{P}^1$  gets smaller, there will be gaugino condensation in  $SU(N_k)$  theory, the decoupled  $U(1)$  will be restored back into the picture.

To see this more precisely, let us consider the case of N D5 brane wrapping  $\mathbf{P}^1$  in the resolved conifold for simplicity (the arguments for the general case is similar). After the transition, the plane M5 curve will be given by  $\Sigma : xy = \zeta$  in the x-y plane which is  $\mathbf{C}^*$ . But it is wrong to use the flat metric for  $\mathbf{C}^*$  here. This M5 was obtained from

two NS, say  $NS_x, NS_y$ , branes by collapsing the  $\mathbf{P}^1$  cycle. Hence for large values of  $x$ , the metric on M5 curve will be like that of  $NS_x$  and for small values of  $x$ , the metric on M5 curve will be like that of  $NS_y$  at large value of  $y$  since two NS branes are smeared out at the intersection point. The metric on M5 is nothing but the induced metric from the flat metric on the x-y plane which is the 11 dimensional metric in our M theory.

Even though for such a metric, the integral  $\int_{\Sigma} d \log x \wedge *d \log x$  is still divergent, now we can regulate the infinity to obtain finite period by putting a cutoff, thus using the same argument for cutoff as in [6]. Because  $|\zeta|$  has dimension 3,  $|x|$  and  $|y|$  have dimensions 3/2. By putting the cutoff at  $|x| = \Lambda_0^{3/2}$ , the integral

$$\begin{aligned} \int_{\Sigma} d \log x \wedge *d \log x &\sim 2 \int_{\Lambda_0^{3/2} > |x| > |\zeta|^{1/2}} dx \wedge d\bar{x} / |x|^2 \\ &= 3 \log \Lambda_0 - \log |\zeta| \end{aligned} \quad (22)$$

becomes finite and these harmonic one forms are exactly captured by the non-compact part  $(\mathbf{C}^*)^n$  of the Jacobian  $J(\Sigma)$ . We observe that the IR divergency of the integral is removed because of  $|\zeta|$  so the only necessary cutoff is in the UV.

Hence the decoupled  $U(1)$  gauge fields will be restored after the transition. In view of this, it seems to be reasonable to assume that there is a MQCD theory associate to the Jacobian  $J(\Sigma)$  which is a  $\prod_{k=1}^n U(N_k)$  gauge theory and is not broken to  $U(1)^n \times \prod_{k=1}^n SU(N_k)$  since the exact sequence (20) does not split in general.

From (22) we can extract the coupling constant for the theory as:

$$\frac{1}{g^2} \sim 3 \log \Lambda_0 - \log |\zeta| \quad (23)$$

which is the expected running of the coupling constant for  $D = 4, \mathcal{N} = 1$  Yang-Mills theory if we replace the cutoff  $\Lambda_0$  by the scale of the gauge theory, denoted by  $\Lambda$ . The coupling constant of  $U(1)$  theory is  $1/N$  of the coupling constant for  $U(N)$  theory.

There is also a discrete  $\mathbf{Z}_2$  symmetry of exchanging  $x$  and  $y$

$$x \rightarrow y, \quad y \rightarrow x, \quad t \rightarrow \zeta^N t^{-1} \quad (24)$$

reverses the orientation of strings stretched between different D4 branes. This  $\mathbf{Z}_2$  symmetry corresponds to the exchange of two factors of  $\mathcal{O}(-1)$  in the  $\mathcal{O}(-1) + \mathcal{O}(-1)$  over  $\mathbf{P}^1$  combined with the involution  $w \rightarrow w'$  on  $\mathbf{P}^1$  which is an exact symmetry of the vacuum.

## 5 Domain Walls and QCD Strings

Confinement is one of the most mysterious and puzzling aspects of QCD. Assuming that  $\mathcal{N} = 1$  MQCD is in the same universality class as QCD [3], we can study some aspects of QCD strings. As a consequence of having  $N$  different vacua after the transition, there can be domain walls separating different vacua. In the large  $N$  limit, they behave as M5 branes where the QCD strings can end [3].

Recall that the vacua are described by  $N$  plane M5 curves:

$$\Sigma_k : \quad t = t_0, \quad xy = \zeta \exp 2\pi i k / N, \quad k = 0, 1, \dots, N-1. \quad (25)$$

A domain wall is a physical situation that for one region of  $x^3$  looks like one vacuum of the theory and for another region of  $x^3$  looks like another vacuum. Here  $x^3$  is one of the three spatial coordinates in  $\mathbf{R}^{1,3}$  and the physics should be independent of the time  $x^0$  and the other two spatial coordinates  $x^1, x^2$ . For convenience, we compactify  $x^3$  direction into a circle  $\mathbf{S}^1$  from now on and use the angular coordinate  $\theta$  for the phase of  $\mathbf{S}^1$ . So we are assuming that we have the same physics at  $x^3 \rightarrow \infty$  and  $x^3 \rightarrow -\infty$ . In M theory, such a physical situation can be realized as a fivebrane that interpolates between the  $N$  plane M5 curves  $\Sigma_k$  describing different vacua. Therefore we consider a fivebrane of the form  $\mathbf{R}^3 \times \mathcal{D}$  where  $\mathbf{R}^3$  is parameterized by  $x^0, x^1, x^2$ , and  $\mathcal{D}$  is a three-surface in the seven manifold  $\tilde{M} = \mathbf{S}^1 \times M$  where  $\mathbf{S}^1$  is parameterized by  $\theta$ ,  $0 \leq \theta < 2\pi$ . We define  $\mathcal{D}$  in  $\tilde{M}$  with fixed  $t$  as

$$t = t_0, \quad xy = \zeta \exp(i\theta). \quad (26)$$

We have a smooth family of holomorphic curves parameterized by  $\theta$  in  $\tilde{M}$ . Since it is locally diffeomorphic to a product of  $\mathbf{R}$  and a Riemann surface, it is supersymmetric. For  $\theta = 2\pi k / N, k = 0, \dots, N-1$ , the three-surface  $\mathcal{D}$  will be a holomorphic curve  $\Sigma_k$  which describes the  $\mathcal{N} = 1$   $U(1)$  supersymmetric Yang Mills with vacua of the glueball superfield  $S$  of the  $SU(N)$  theory. Hence this is the domain wall connecting  $\mathcal{N} = 1$   $U(1)$  theory with all possible different vacua.

Recall that the vacua are described by  $N$  plane M5 curves  $\Sigma_k$ :

$$t = t_0, \quad xy = \zeta \exp(2\pi i k / N), \quad k = 0, \dots, N-1 \quad (27)$$

in  $M$ . We now define QCD strings as open oriented onebrane  $C_k$  in  $M$  with fixed  $t$ :

$$\begin{aligned} t &= t_0, \\ x &= t_0^{1/N} \exp(2\pi i(k + \sigma)/N), \\ y &= \zeta t_0^{-1/N}. \end{aligned} \quad (28)$$

The curve  $C_k$  connects two vacua  $\Sigma_k$  and  $\Sigma_{k+1}$ . As a topological object, they are classified as the element of a relative homology  $H_1(M, \cup \Sigma_k, \mathbf{Z})$ . The relative homology fits into a long exact sequence:

$$\dots \rightarrow H_1(\cup \Sigma_k, \mathbf{Z}) \rightarrow H_1(M, \mathbf{Z}) \rightarrow H_1(M, \cup \Sigma_k, \mathbf{Z}) \rightarrow H_0(\cup \Sigma_k, \mathbf{Z}) \rightarrow \dots \quad (29)$$

Using this sequence and topological properties of  $\Sigma_k$  and  $Y$ , it can be shown that

$$H_1(M, \cup \Sigma_k, \mathbf{Z}) \cong \mathbf{Z}^n. \quad (30)$$

The  $N$  of  $C_k$  will annihilate as an element of  $H_1(M, \mathbf{Z})$  i.e. they can be detached from  $\cup \Sigma_k$  as the boundaries add up to zero and will form a long closed string in  $M$  and can be deformed into a point. By construction, all of the QCD strings lie entirely in the domain wall  $\mathcal{D}$  and hence stable.

Recently, the BPS domain wall of supersymmetric Yang-Mills theory for arbitrary gauge theory has been studied [33]. For  $SU(N)$  group, the M-theory formulation on  $\mathbf{G}_2$  holonomy geometry has been used. It would be desirable to see the results of [33] from the MQCD set-up as above.

## 6 Brane Construction from Fluxes

So far we've studied the large  $N$  transition purely from M-theory point of view. What happens in string theory? In this section we shall provide an intuitive understanding of the large  $N$  duality using type IIA brane constructions. But before we dwell into that we need some more details on the conifold. As was shown in the previous sections, the T dual of type IIB theory on a conifold can be regarded as two NS5 branes at a point  $x^6 = x^7 = 0$ . The distance between the two NS5 branes along  $x^7$  is given by the resolution of a conifold and the distance along  $x^6$  is given by the value of  $B_{NSNS}$  field on the vanishing two cycles of the base sphere in the T-dual model. The latter can be shown easily if we have some D4 branes along the compact direction  $x^6$  and also stretched along  $x^{0,1,2,3}$ . The distance between the NS5 branes determines the  $3+1$   $d$  gauge coupling which in turn is determined by the  $\int B_{NSNS}$  over the two cycle  $S^2$  of the base of the conifold. If we have both the situation, namely, a resolved conifold with some  $B_{NSNS}$  flux through the two cycles then the T-dual picture is given by two NS5 branes separated along a distance  $z_0$  in a complex  $z = x^6 + ix^7$  plane.

The  $S^3$  of the base is a Hopf fibration of a circle on a sphere  $S^2$ . The circle, whose coordinates we specify as  $\psi$  is non trivially twisted over the base  $S^2$ . This twist can be seen to come from the NS5 brane charges in the T-dual picture. To make this

connection precise we first make an identification of the brane coordinates  $x^n$  with the conifold coordinates  $\theta_i, \phi_j, \psi$ . For simplicity we can have the following identifications:

$$dx^{4,8} \rightarrow \sin \theta_{1,2} d\phi_{1,2}, \quad dx^{5,9} \rightarrow d\theta_{1,2} \quad (31)$$

In these units the magnitude of the two  $B_{NSNS}$  fields  $B_i$  satisfy<sup>4</sup> the equation

$$\frac{1}{\sin \theta_i} \frac{\partial}{\partial \theta_i} (\sin \theta_i B_i) = \text{constant} \quad (32)$$

whose solutions are given by  $\cot \theta_1$  and  $\cot \theta_2$ . Thus the non trivial twist of the cycle  $\psi$  on the base is governed by the metric

$$ds^2 = (d\psi + \cos \theta_1 d\phi_1 + \cos \theta_2 d\phi_2)^2 \quad (33)$$

Here  $\psi = x^6$ . There are also two  $S_i^2$ ,  $i = 1, 2$  whose metric is given by the usual polar coordinates

$$\sum_{i=1}^2 (d\theta_i^2 + \sin^2 \theta_i d\phi_i^2) \quad (34)$$

However it turns out that in homology, the  $S^2$  factor in  $S^2 \times S^3$  is the difference of the two  $S^2$ 's parameterized by  $\theta_1, \phi_1$  and  $\theta_2, \phi_2$  respectively. To see this observe that we can write two 2-forms

$$\sin \theta_1 d\theta_1 d\phi_1 \pm \sin \theta_2 d\theta_2 d\phi_2 \quad (35)$$

both of which live on the two  $S^2$  factors and are independent of the  $U(1)$  fiber.

Both of these 2-forms can be written formally as exact forms, namely the above expressions are equal to

$$d(\cos \theta_1 d\phi_1 \pm \cos \theta_2 d\phi_2) \quad (36)$$

However, the expressions in brackets above are ill-defined when any of the  $\theta_i$  is equal to 0 or  $\pi$ , since in that case we are at the north or south pole of one of the 2-spheres and the coordinate  $\phi_i$  is undefined there. However, the term in the  $+$  sign can be modified to:

$$d(d\psi + \cos \theta_1 d\phi_1 + \cos \theta_2 d\phi_2) \sim de^\psi \quad (37)$$

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<sup>4</sup>There are various  $B$ -fields in the model. The  $B_i$ 's discussed here are the sources of the two NS5 branes. We take  $B_1 \equiv B_{64}$  and  $B_2 \equiv B_{68}$  only as it is easy to show that the other components can be gauged away. The  $\int B_{NSNS}$  field discussed above is a flux through the two cycle.

where  $e^\psi$  is one of the five vielbeins given in appendix (A) of Ref. [36], and is globally defined because  $\psi$  is allowed to have gauge transformations. It follows that the term with the  $+$  sign in eq. (35) is genuinely exact, leaving the one with the minus sign as the representative of the second cohomology. From this the above claim follows.

Now, with the above back-ground, let us try to understand this duality from our brane construction. However there is a small subtlety here. The brane construction that we use here for deformed conifold is an approximation because the derivation relies on the fact that some of the directions are actually de-localized. Therefore the brane construction for deformed conifold, in this section, should be regarded as for de-localized branes. The exact brane construction is been derived in sec. 2 which takes only localized branes. But since for completely localized branes we do not know how to calculate the exact supergravity metric we, for the purpose of this section only, take the approximate framework. Also, as it will be clear soon, we need only the brane construction for the conifold with  $S^3 \rightarrow 0$ .

We start with the deformed conifold with RR flux on the  $S^3$ . For simplicity we shall assume that the total flux is constant. Now let us shrink the  $S^3$  cycle to zero size. In the limit when the cycle has shrunk to zero size then the resulting manifold is a singular variety and is our old friend conifold.

The T-dual of this process is known. As we discussed in sec. 2, the T-dual of a deformed conifold is a set of two intersecting NS5 branes on a curve  $xy = \mu$  and in the limit  $\mu \rightarrow 0$  it is nothing but two NS5 branes intersecting at a point (which is, of course, a  $3 + 1$  dimensional plane). But now this is not enough. Recall that in the IIB picture we have a large  $H_{RR}$  flux at the conifold point. The  $H$  field is on  $S^3$  and has components  $H_{\psi\mu\nu}$  where  $\mu, \nu$  are coordinates on the base sphere. The T-dual of  $H_{\psi\mu\nu}$  gives us  $F_{\mu\nu}$  in type IIA theory where  $F = dA$ ,  $A$  being the RR gauge field. Therefore we have  $N$  units of flux at the point where the two NS5 branes meet. It is important to ask what directions do the  $\mu, \nu$  coordinates span. The coordinates  $\mu, \nu$  are on the base of  $S^3$ . The cohomology of the  $S^3$  is generated by the third cohomology  $H^3(T^{11})$  as:

$$e^\psi \wedge e^{\theta_1} \wedge e^{\phi_1} - e^\psi \wedge e^{\theta_2} \wedge e^{\phi_2} \quad (38)$$

where we have already defined  $e^\psi$  and  $e^{\theta_i}, e^{\phi_i}$  are defined in terms of the conifold variables as:

$$e^{\theta_i} = d\theta_i, \quad e^{\phi_i} = \sin \theta_i d\phi_i, \quad i = 1, 2 \quad (39)$$

Therefore  $\mu, \nu$  span all the four directions  $x^{4,5,8,9}$  and hence  $F$  will have components along both the spheres  $x^{4,5}$  and  $x^{8,9}$ .

When the  $S^3$  is shrunk to zero, we have, in the T-dual model, the two NS5 branes at the point  $x^6 = x^7 = 0$  and the directions  $x^{4,5}$  and  $x^{8,9}$  of the NS5 branes wrapped on

vanishing spheres. This is a singular configuration (with flux per unit area approaching infinity) and because of this a *four-brane* is created by a mechanism similar to the Hanany – Witten effect[27]. To understand this let us recall some basic facts about the NS5 brane.

On the world volume of a NS5 brane there propagates a chiral  $(2, 0)$  tensor multiplet whose fields are  $(B_{ij}^+, 5\phi)$ .  $B^+$  is an anti-self dual two forms on the NS5 brane whose coordinates are  $\sigma^{i,j}$  and  $\phi_i, i = 1, \dots, 5$  are the five scalars. Two out of these five scalars are periodic. A simple way to see this would be as follows [37]:

If we T-dualize orthogonal to a de-localized NS5 brane of type IIA we shall get a configuration of Kaluza-Klein (KK) monopole in type IIB theory. As have been discussed in many previous works, the KK monopole supports a normalizable harmonic form which is anti self-dual. The four form of type IIB when reduced over this two-form gives rise to the two form  $B_{\mu\nu}^+$  of the NS5 brane. The two  $B$  fields of type IIB  $B_{RR}$  and  $B_{NSNS}$  give the two periodic scalars on the NS5 brane. The other three scalars come from the translational zero modes of the KK monopole.

The background gauge field couples to the world volume of the NS5 brane through the following coupling:

$$\int A \wedge *d\phi = \int F \wedge C_4 \quad (40)$$

where  $dC_4 = *d\phi$  is the six dimensional dual of a world volume periodic scalar. From type IIB point of view this coupling can be motivated from the kinetic term of the  $H$  fields.

Because there is a background expectation value of the gauge field, this will induce a source  $\int C_4$  on the world-volume of the NS5 brane. A generic source will break all supersymmetry on the world volume. The susy preserving source will tell us that we have a *four-brane* stretched between the two NS5 branes.

Along what direction should this D4 stretch? It can be argued that the four- brane should be along  $x^{0,1,2,3}$  and  $x^7$ . Observe that in our model there are two orthogonal directions  $x^6$  and  $x^7$ . The limit where  $S^3$  is of zero size is, as we discussed above, when the  $x^{4,5}$  and  $x^{8,9}$  spheres are of zero size and the branes are on top of each other. In the type IIB T-dual model the conifold transition will tell us that we go to a non-singular configuration by growing a *two-sphere*. The branes should therefore be now separated along  $x^7$  because this process is the T-dual of resolution in the conifold. Thus the final configuration will be a D4 stretched between two NS5 branes separated along  $x^7$ , which is precisely the configuration that we discussed in detail in the previous sections. But this is nothing but the T-dual of a resolved conifold with a D5 wrapping the  $S^2$ ! Observe

that we have arrived at this duality by a transition via a singular configuration – a conifold transition.

In the discussion above we gave a physical picture to motivate the large N duality of Vafa. One last thing is to see how from M-theory this transition could be understood. Recall that in M-theory we start in “reverse” order, i.e we start with a configuration of a *single* M5 brane coming from a N D4 branes between two orthogonal NS5 branes in type IIA theory. After the transition the space M5 branes become N “planar” M5 branes which are located at a particular position in  $x^{10}$ . What does an observer in  $d = 10$  sees? The two M5 which intersect at a point in  $x^{10}$  will appear as two intersecting NS5 branes satisfying the equation  $xy = m\zeta$  where  $m$  is a constant factor. The bending along the  $x^{10}$  direction, which gives non trivial components of the metric, will appear as gauge fields  $A_\mu$  in 10 dimensions. Again this is precisely the brane configuration we started with in the beginning of this section.

Finally, what about the  $U(1)$  fields? In the above discussions we have motivated the  $U(1)$  fields as the one coming from the  $(2,0)$  fields of M5 branes. Also we know from the large N duality that the  $U(1)$  gauge fields have their origin from type IIB four form reduced over holomorphic three form of the Calabi-Yau. How are they related? As we discussed in some details the two form of M5 brane has its origin from the four form of type IIB. Using various dualities of NS5 brane to a Taub-NUT space and two intersecting NS5 branes to a conifold one can easily show that the two  $U(1)$  are the same. This is how the transition from  $U(N)$  theory on the brane to  $U(1)$  theory on the bulk can be seen.

Before we end, one very interesting direction is to study the interpolation between the supergravity solution corresponding to  $D5$  branes wrapped on the  $\mathbf{P}^1$  cycle of the resolved conifold and the one corresponding to fluxes on the  $\mathbf{S}^3$  cycle of the deformed conifold <sup>5</sup>. On one side we have the results of [38, 39] for  $D5$  branes wrapped on  $\mathbf{P}^1$  and on the other side we have results concerning the deformed conifold without fluxes [40, 41]. The work towards an interpolating solution is in progress.

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